

140 online μάθημα

13/5/2020

Άσκηση 5.2:

$$Q(b_0, b_1) = \sum_{i=1}^n (y_i - b_0 - b_1(x_i - \bar{x}))^2$$

$$\frac{\partial Q}{\partial b_0} = -2 \sum_{i=1}^n [y_i - b_0 - b_1(x_i - \bar{x})]$$

$$\frac{\partial Q}{\partial b_1} = -2 \sum_{i=1}^n (x_i - \bar{x}) [y_i - b_0 - b_1(x_i - \bar{x})]$$

$$\text{Given } \sum y_i - b_1 \sum (x_i - \bar{x}) = n \hat{b}_0 \Rightarrow \hat{b}_0 = \bar{y}$$

$$\sum (x_i - \bar{x}) y_i - \hat{\beta}_0 \sum (x_i - \bar{x}) = \hat{\beta}_1 \sum (x_i - \bar{x})^2$$

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2} = \frac{\sum (x_i - \bar{x}) (y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

Abkürz. 5.3:

$$(i) \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x}) (y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\bar{x} = 0 = \frac{\sum x_i}{n}, \quad \bar{y} = \frac{\sum y_i}{n} = \frac{102}{11} = 9.2727$$

$$\sum (x_i - \bar{x})^2 = \sum x_i^2 = 110$$

$$\sum x_i y_i = 158, \quad \sum x_i^2 = 110$$

$$\hat{\beta}_1 = \frac{158}{110} = 1.441 \quad \hat{\beta}_0 = 9.27$$

$$\hat{y} = 9.27 + 1.44x$$

$$(ii) SS_{tot} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n} \Rightarrow SS_{tot} = 248.18$$

$$SS_{reg} = \hat{\beta}_1^2 \sum (x_i - \bar{x})^2 = \hat{\beta}_1 \sum (x_i - \bar{x}) (y_i - \bar{y}) \Rightarrow SS_{reg} = 226.25$$

$$SS_{rer} = SS_{tot} - SS_{reg} = 21.93$$

ANADIA

$$SS_{reg} = 226.25 \quad 1 \quad MS_{reg} = 226.25$$

$$SS_{rer} = 21.93 \quad 9 \quad MS_{rer} = 2.36$$

$$SS_{tot} = 248.18 \quad n-1 = 10$$

$$F = 96.18 > F_{0.05, 1, 9} = 5.12 \quad \text{Απορριπτεται}$$

$$(iii) \hat{\beta}_1 \pm t_{\alpha/2, n-2} \frac{S}{\sqrt{\sum (x_i - \bar{x})^2}}$$

$$\text{Προβωπή } S^2 = N S_{\text{res}} \Rightarrow S = \sqrt{2.36}$$

$$t_{0.025, 9} = 2.262$$

$$\text{Άρα } \hat{\beta}_1 \in [1.11, 1.77]$$

$$(iv) \hat{Y}_0 \pm t_{\alpha/2, n-2} S \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

$$\hat{Y}_0 = 9.27 + 1.44 \cdot 3$$

$$\text{Προκύπτει } E(Y_0) \in [12.15, 15.03]$$

$$(v) R^2 = \frac{226.95}{243.1} * 100\% = 91.42\%$$

Οι ακρότητες 5.4, 5.5, 5.6, 5.7, 5.8 δεν έχουν καμία διαφοροποίηση με την 5.3

Οι ακρότητες 5.9, 5.10 εκτός εξεταστέας ύλης.

Άσκηση: Έστω  $Y_i = \beta X_i + \varepsilon_i$ ,  $i = 1, \dots, n$  μοντέλο παλινδρόμησης για το οποίο ισχύει ότι  $E(\varepsilon_i) = 0$ ,  $\text{Var}(\varepsilon_i) = 6^2 X_i$ . Να βρεθεί ο Ε.Ε.Τ του  $\beta$  και ν.δ.ο. είναι ακερόρητος.

Υπόδειξη: Η διακύμανση δεν είναι σταθερή αφού αλλάζοντας το  $X_i$  αλλάζει το  $\text{Var}$ .

$$\text{Var}(\varepsilon_i) = 6^2 X_i$$

$$\text{Var}(\varepsilon_i) = 6^2 \quad (\text{Var}(aY) = a^2 \text{Var} Y)$$

$X_i$

$$\text{Var}\left(\frac{\varepsilon_i}{\sqrt{X_i}}\right) = 6^2 \Rightarrow \frac{Y_i}{\sqrt{X_i}} = \beta \sqrt{X_i} + \frac{\varepsilon_i}{\sqrt{X_i}}$$



$$\Rightarrow Y_i' = \beta X_i' + \varepsilon_i'$$

$$Q = \sum (Y_i' - \beta X_i')^2$$

$$\frac{\partial Q}{\partial \beta} = 0$$

$$\beta$$